Cautionary Notes on Determining Terminal Value in the DCF Model

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TERMINAL VALUE

- Terminal value is the dominant component of most DCF valuations
  - With 5-year projections, terminal value usually accounts for 70% or more of the aggregate value
- This presentation will examine several factors that impact terminal value and discuss how to address them
  - The final year of the projection
  - The trend toward using lower long-term growth rates
  - The "perpetual" growth rate and firm mortality
  - The use of multiples for terminal value
  - The relationship between capital expenditures and depreciation
  - The appropriate treatment of amortization

The Final Year of the Projection

Due Diligence

- Terminal value is a direct function of the final year of the projection underlying the DCF analysis
- The analyst should conduct due diligence to determine the reasonableness of the projection and the underlying assumptions
- Normalizing adjustments should be made to adjust inputs that will not grow in parallel with revenues and free cash flow

NORMALIZATION

- Some normalizing adjustments depend on the purpose of the valuation, *e.g.*:
  - If the company is being valued as a going-concern under its current management, no normalizing adjustments are needed for such items as excess compensation or management perks because these would be expected to continue
  - If a company is being valued at financial control value, normalizing adjustments for excess compensation and management perks would be appropriate because these would be changed by a buyer

STEADY STATE

- In applying a growth model, the analyst should consider whether the company has reached a "steady state" of growth by the final year of the projection
- If in the final year of the projection, the company is still growing at a faster rate than its expected long-term growth rate, the use of a multi-stage model is necessary
- Some companies, such as mining and oil & gas production, may have negative long-term growth rates in their "steady state"

Trend Toward Using Lower Long-Term Growth Rates

Trend Toward Using Lower Growth Rates

- Common practice for determining terminal value has been to assume that a company's perpetual growth rate should be close to the expected long-term growth of the economy
- In two recent studies,* I have examined the discount rates used by investment bankers in connection with publicly-disclosed fairness opinions
- Data from these studies indicate that since the 2008 recession, investment bankers have tended to use lower growth rates to calculate terminal value

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* Cited articles, as well as other selected articles, are listed in the Bibliography appended to this presentation

Trend toward Lower Growth Rates

Midpoints of Growth Rates in Growth Models for Fairness Opinions						
	Cash Acquisitions: 9/2007–8/2008		Cash Acquisitions: 9/2010–8/2011		Stock-for-Stock Mergers: 2009–14	
Median	3.0%		2.5%		2.0%	
Mean	3.4%		2.9%		2.0%	
	Number	Percent	Number	Percent	Number	Percent
Less than 1%	0	0.0%	6	9.0%	20	16.4%
1%	1	2.0%	4	6.0%	13	10.7%
>1% and <2%	2	3.9%	3	4.5%	16	13.1%
2%	7	13.7%	13	19.4%	19	15.1%
>2% and <3%	9	17.6%	11	16.4%	14	11.1%
3%	12	23.5%	16	23.9%	28	22.2%
>3% and <4%	4	7.8%	5	7.5%	6	4.8%
4%	4	7.8%	5	7.5%	3	2.4%
More than 4%	12	23.5%	4	6.0%	7	5.7%
Total	51	100.0%	67	100.0%	122	100.0%

The “Perpetual” Growth Rate and Firm Mortality

The Perpetual Growth Assumption

- In the customary DCF valuation, it is assumed that a mature company will survive and will grow at a constant rate in perpetuity

- This assumption is invalid for two reasons:
  - The impact of corporate mortality
  - The impact of decelerating company growth due to economic changes and/or obsolescence

- The constant perpetual growth assumption can result in overstated values

Changes in Tenure of Top 500 Companies

- Companies in the 1958 S&P 500 were in the index for an average of 61 years (based on seven year rolling averages)
- By 1980, the average tenure had declined to about 25 years
- Now the average tenure is about 18 years
- Over the decade to 2012, about half the S&P 500 was replaced
- Only 61 companies that were in the 1955 Fortune 500 remain in the 2015 Fortune 500

Companies Are Dropped from Indices for Various Reasons

- Examples of companies dropped from S&P 500: 2001-2012

American Airlines:	restructured in bankruptcy
Anheuser-Busch:	acquired by InBev
Bear Stearns:	insolvent, taken over by JP Morgan
Eastman Kodak:	restructured in bankruptcy
Enron:	bankrupt, ceased operations
Global Crossing:	restructured in bankruptcy
Lehman Brothers:	bankrupt, ceased operations
May Dept. Stores:	acquired by Macy’s
Maytag:	acquired after material reduction in sales
NY Times:	slow growth
Palm:	sales decline and financial problems
Radio Shack:	financial problems
Sears:	restructured in bankruptcy
Toys “R” Us:	taken private in LBO
Wendy’s:	merger

What Causes This Attrition?

- What are the reasons for this attrition?
  - Some companies are absorbed in mergers and acquisitions
  - Some companies grow at slower rates and are replaced by faster-growing companies
  - Some companies have financial problems that slow or reverse their growth
  - Some companies are restructured in bankruptcy
  - Some companies cease operations and die

Young Firms Have the Greatest Mortality Risk

- According to the U.S. Bureau of Labor Statistics, about 50% of all new businesses are still operating after 5 years, and about two-thirds of the survivors are still in business after another 5 years
- As firms grow older and larger, the risk of failure in any given period declines
- For companies that mature and become listed, Loderer, Neusser and Waelchli conclude that the frequency of corporate failure falls "from about 3% in early years [after listing] to 0.3% before companies get to be 75"

THE MORRIS ARTICLE

"LIFE AND DEATH OF BUSINESSES: A REVIEW OF RESEARCH ON FIRM MORTALITY"

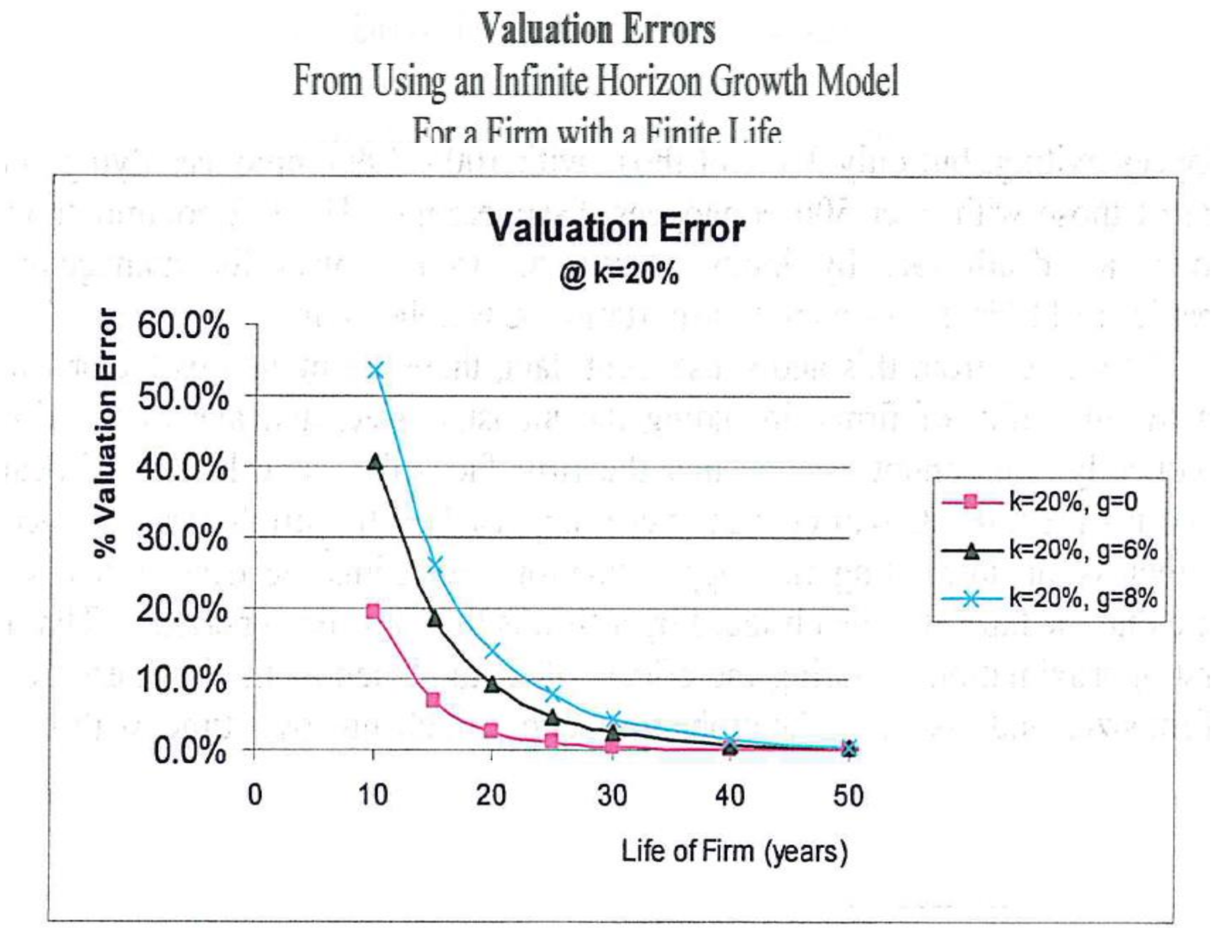
- Prof. James Morris (2009a) notes:

  [R]elatively little attention is given to expected life in the valuation literature and in the valuation methods used by practitioners. . . . The constant growth model is as accurate as the assumptions on which it is based: an infinite horizon and growth that is expected to be the same rate every period forever. If the firm's circumstances do not fit these assumptions, the model can lead to an inaccurate valuation. How inaccurate depends on how far the assumptions depart from reality.

- This thoughtful article discusses firm survival and mortality, examines available data, and addresses the impact of a constant growth assumption on corporate valuation

IMPACT OF FINITE LIFE ASSUMPTION

- Morris calculated the impact of a finite life assumption vs. an infinite life assumption



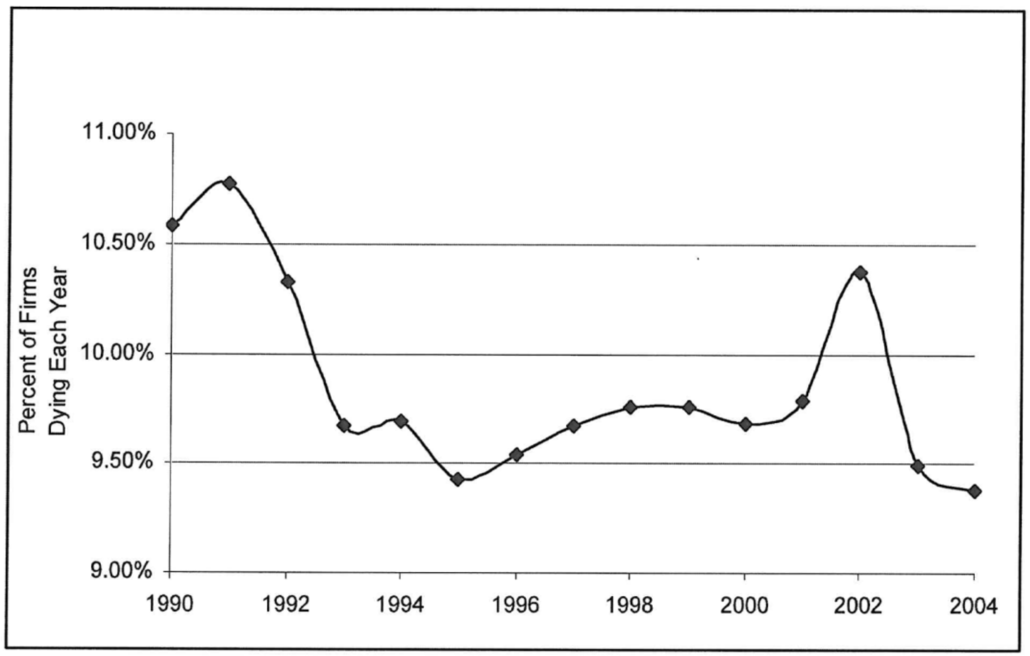
k = discount rate

g = growth rate

BUSINESS MORTALITY DATA

- He examined U.S. government data as to business mortality

Death Rates Of U.S. Business Firms



Source: U.S. Small Business Administration, Office of Advocacy.
sba.gov/advo/research/data, 2007

STUDIES OF SURVIVAL RATES

- Morris also reviewed prior studies of survival rates

Summary of Survival Rates

	Cumulative Survival Rate			
Authors	4 years	5 years	10 years	Source of Data
Dunne, Roberts & Samuelson (1989)		43.4%	26.3%	U.S. Census of Manufacturers 219,754 mfg. plants; 1963 - 1982
Audretsch (1991)	63.1%		35.4%	Small Business Database 11,154 manufacturing firms; 1976 - 1986
Agarwal & Audretsch (2001)		66.1%	48.7%	Thomas Register of Manufacturers 3,431 firms, 1906 - 1991
Exponential Model	67.4%	61.1%	37.4%	Small Business Administration 5.7 million firms in all sectors, 2003 - 04

Impact of Mortality Risk

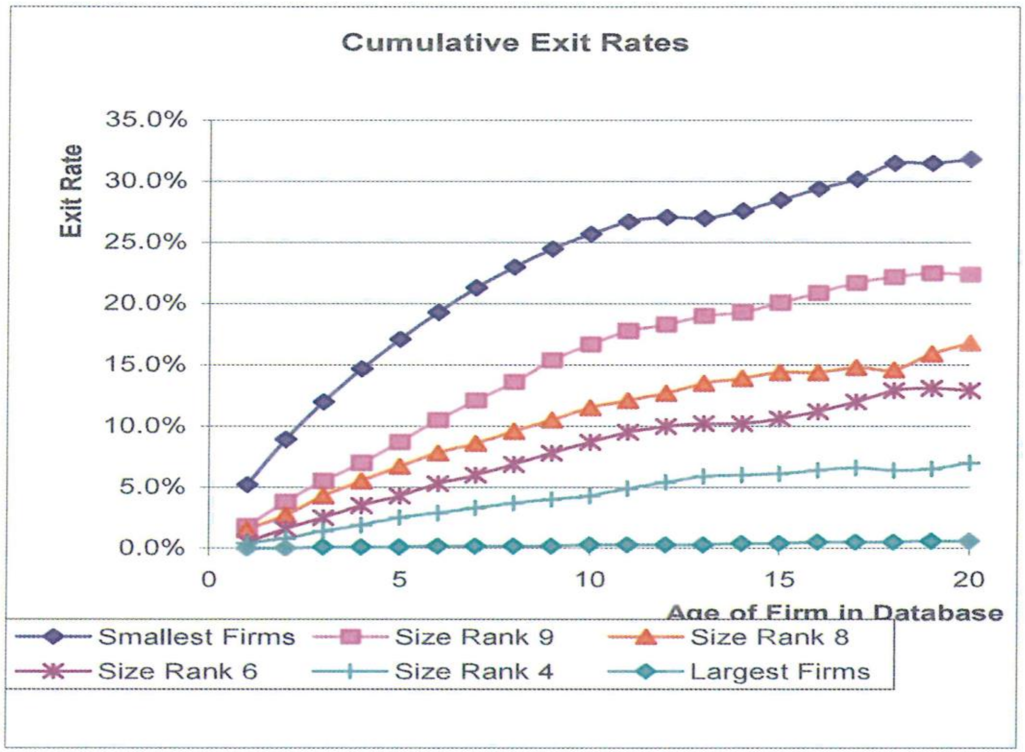
- If the risk of failure in any given year is 1% and is constant year to year, the cumulative risk of failure within 15 years is 14%
- If the risk of failure in any given year is 0.6% and is constant year to year, the cumulative risk of failure within 25 years is 14%
- If the risk of failure in any given year is 1% and is constant year to year, the cumulative risk of failure within 25 years is 22%

Cumulative Risk of Failure				
Per year	**0.4%**	**0.6%**	**0.8%**	**1.0%**
10 years	3.9%	5.8%	7.3%	9.6%
15 years	5.8%	8.6%	10.8%	14.0%
20 years	7.7%	11.3%	14.2%	18.2%
25 years	9.5%	14.0%	17.4%	22.2%

FIRM MORTALITY IS INVERSELY RELATED TO SIZE

- Morris points out that firm mortality is a function of size

Exit Rates for Firms Due to Unfavorable Mortality
For Selected Size Categories



Source: Queen & Roll (1987), p. 24.

Decile 1 = Largest Firms
Decile 10 = Smallest Firms
(by market value of equity)

Effect of Growth and Discount Rates

- This magnitude of the impact of firm mortality on firm value is a function not only of the mortality risk, but also of the growth rate and the discount rate

  - The impact on value increases at higher growth rates

  - The impact on value decreases at higher discount rates

THE GORDON GROWTH FORMULA

- The standard formula for calculating terminal value using the Gordon growth model is

$$PV = \frac{CF\,(1 + r)}{r - g}$$

PV = present value of future cash flows

CF = free cash flow in final year of projection

r = discount rate

g = long-term growth rate

Adjusting the Growth Formula for Risk of Failure

- How can the Gordon growth formula be adjusted to reflect the risk of failure?
- Prof. Sherrill Shaffer (2006) proposes adjusting the formula for the probability (p) that "the asset may irreversibly default (*i.e.*, the issuing company may fail) in any given year":

$$PV = \frac{CF\,(1 + r)\,(1 - p)}{r + p - g\,(1 - p)}$$

Discount Rates and Growth Rates Adjusted for Risk of Failure

- He solves this formula to determine

  R – the discount rate adjusted for p, and

  G – the growth rate adjusted for p

$$R = \frac{p\,(1+r)^2}{1+g-p\,(r+g+2)}$$

$$G = \frac{rg\,(1-p)}{r+p}$$

How Can the Risk of Failure Be Determined?

- Shaffer (2007) wrote:

  The simplest way to estimate p is to use historical average business failure rates, which are widely available. . . . Recognizing that different industries sometimes exhibit very different failure rates, sector-specific failure rates may be more appropriate. . . .

  A more detailed and forward-looking approach would involve statistical models predicting firm-specific probabilities of failure, based on current financial data for each firm and calibrated using historical linkages between financial ratios and subsequent failure.

DAMODARAN'S FORMULA

- Prof. Aswath Damodaran proposes a formula for adjusting enterprise value for the risk of financial distress

$$AV = PV \times (1-p) + DSV \times p$$

AV = adjusted value

PV = unadjusted present value based on DCF

DSV = distressed sales value

p = probability of distress

Damodaran: Determining the Probability of Distress

- Damodaran also posits that statistical techniques can be applied to historical data to determine the probability of distress as a function of observable variables
  - He notes that factors such as high debt ratios and negative cash flows increase the risk of failure
- He also points out that bond ratings and the historical relationship between ratings and defaults can be used to estimate the mortality risk
  - This approach is necessarily limited to companies with published bond ratings

THE SAHA – MALKIEL ARTICLE

"VALUATION OF CASH FLOWS WITH TIME-VARYING CESSATION RISK"

- Atanu Saha and Burton Malkiel (2012) point out:

> *Because CAPM-based discount rates only account for market risk, valuation models may greatly underestimate the discount rate . . . in settings where the idiosyncratic risk of the cash flows matters. This is especially so in cases where there is a significant probability that the future stream of cash flows may completely cease. This is a risk that the CAPM ignores because that model assumes it is a risk that can be diversified away. . . . [W]e believe that an additional adjustment to the discount rate is warranted to account for cash flow cessation probability, in settings where such a possibility is not immaterial.*

Saha–Malkiel Framework

- They develop a framework for calculating present value "when cash flows have a finite probability of cessation at each period"
- They "present a simple formula for the cessation risk-adjusted discount"
- They "then extend the analytical framework to allow for the possibility of a time-varying cessation risk"

SAHA–MALKIEL FORMULAS

- The Saha–Malkiel formula with a constant "cessation risk" is the same as Shaffer's formula
- They then create a formula based on the assumption that the cessation risk declines as the firm ages
  - This complex formula is a further helpful step for adjusting valuations to reflect mortality risk

Valuators Should Consider Whether to Adjust for Mortality Risk

- Today's general practice of using a perpetual growth rate calculating terminal value needs to be reexamined
- Adjustments for firm mortality or for the risk of decelerating growth should be considered
  - For companies with a low mortality risk, the impact may be immaterial
  - Venture capitalists commonly account for the substantial possibility that a start-up company may not succeed by using discount rates of 35% or more

Further Empirical Research Is Needed

- The valuation community – and the academic community – should consider how to quantify the risks not only of mortality but also of declining (or negative) long-term growth
  - How can these risks be reflected in higher discount rates and/or lower long-term growth rates?
- Further empirical research into firm decline and mortality is necessary to develop the appropriate risk premiums

Using Multiples to Calculate Terminal Value

Terminal Value Is Sometimes Calculated Using Multiples

- The use of exit multiples for determining terminal value is criticized by academics and other commentators for intermixing two different valuation approaches

- Shannon Pratt explains:

  The market multiple brings a major element of the market approach into the income approach.*

- Nonetheless, multiples (primarily of EBITDA) are commonly used by investment bankers to calculate terminal value

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* Shannon P. Pratt, *Valuing a Business*, 5th Ed. (McGraw Hill, 2008), p. 220

Investment Banks Use Multiples More Often than Growth Models

- My study of valuation methods used for fairness opinions in U.S. cash acquisitions showed that 65½% used multiples and only 41% used a growth model (6½% used both)
- My forthcoming study of valuation methods used for fairness opinions in U.S. stock-for-stock mergers (2009–2014) shows:

	Financial Institutions	Other Companies	Total
Multiples only	93%	59½%	72%
Growth model only	5%	37½%	25%
Both	2%	3%	3%

Multiples Should Be Normalized

- A common error in using multiples for determining terminal value is to assume that multiples in the terminal year will be at the currently prevailing level
- When the current multiples reflect optimistic growth expectations, the use of current multiples to calculate terminal value causes overstatement of terminal value
- If an exit multiple is used for terminal value, it should be normalized to reflect the “reversion to the norm” as the company’s growth tends toward its long-term growth rate

Sanity Checks on Terminal Values

- The practitioner who determines terminal value using multiples of EBITDA should calculate the implied growth rate and consider whether the result is reasonable
- Similarly, the practitioner who uses a growth model should examine the implied multiples of EBITDA and net income based on the calculated terminal value
- If the multiple-based terminal value implies a unrealistic growth rate (or if a growth model's implied multiples of terminal value are materially inconsistent with projected future multiples), the practitioner should reexamine the underlying assumptions

The Relationship Between Depreciation and Capital Expenditures

A Common Error

- When calculating terminal value in the Gordon growth model, it has been common practice for valuators to assume that depreciation equals capital expenditures in perpetuity
  - In fact, due to growth and inflation, ***capex must be greater than depreciation*** in a growth model
- A common error is to assumes that capex = depreciation
  - Many analyses even have capex < depreciation in perpetuity!
- Understating capex necessarily results in overstated terminal values

Most Practitioners Assume That Depreciation = Capex

- A 2015 survey by Jim Hitchner published in his bi-monthly *Financial Valuation and Litigation Expert*, valuators in a webinar audience were asked:

  How do you typically handle depreciation and cap ex when calculating cash flows?

- The responses were:
  - Capex less than depreciation[!]: 6%
  - The same or very similar: 55%
  - Capex greater than depreciation: 38%

FCF for Terminal Value Should Be Normalized

- The analyst must always review projected capex and depreciation in the terminal year to determine whether normalizing adjustments to FCF are needed
  - Although capital expenditures in any given year can be less than depreciation, a growing company's ***normalized*** capex should exceed its depreciation
  - Equipment costs and evolving technology costs may affect the relationship of the depreciation rate to the growth rate
    - To the extent that new equipment is less expensive or more efficient, the ratio of capex to depreciation may decrease
    - If a single-facility company built and equipped a factory, depreciation could exceed capex until major new investments are required

5-YEAR STRAIGHT LINE DEPRECIATION

- Example: a company depreciates its assets on a straight-line basis over a five-year period to zero residual value and is growing at 5% annually

5 Year Straight Line Depreciation with 5% Growth			
Year Purchased	Capital Expenditures	Depreciated in 2022	
		%	Amount
2017	1,000.0	10%	100.0
2018	1,050.0	20%	210.0
2019	1,102.5	20%	220.5
2020	1,157.6	20%	231.5
2021	1,215.5	20%	243.1
2022	1,276.3	10%	127.6
			1,132.8

- Capex in year 6 is 112.7% of depreciation [1,276.3 ÷ 1,132.8]

5-YEAR DOUBLE DECLINING DEPRECIATION

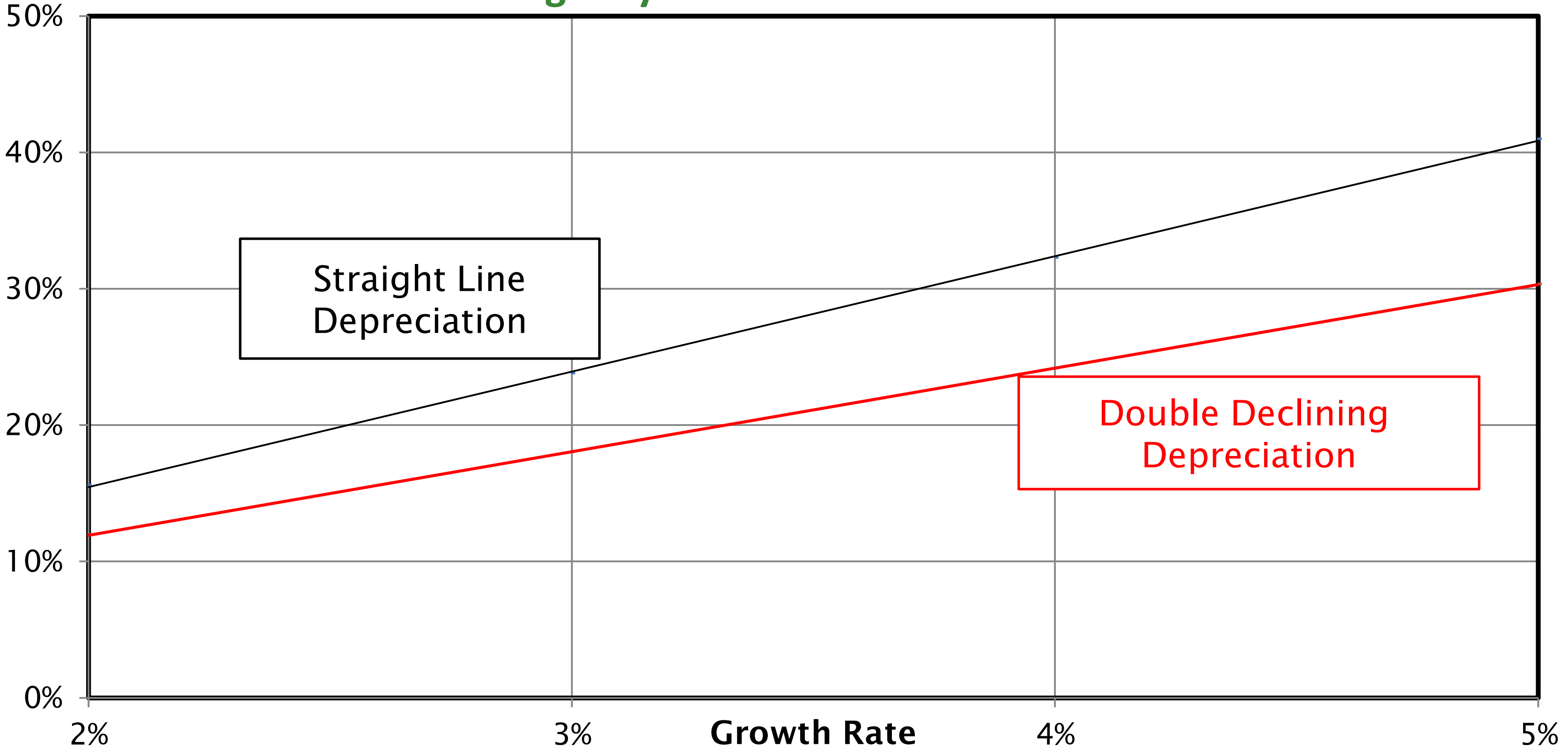
- Five-year double declining depreciation to zero residual value

5 Year Double Declining Depreciation with 2% to 5% Growth								
	2% Growth		3% Growth		4% Growth		5% Growth	
Year	Capex	Depreciated in 2022	Capex	Depreciated in 2022	Capex	Depreciated in 2022	Capex	Depreciated in 2022
2017	1,000	57.6	1,000	57.6	1,000	57.6	1,000	57.6
2018	1,020	117.5	1,030	118.7	1,040	119.8	1,050	121.0
2019	1,040	119.9	1,061	122.2	1,082	124.6	1,103	127.0
2020	1,061	203.8	1,093	209.8	1,125	216.0	1,158	222.3
2021	1,082	346.4	1,126	360.2	1,170	374.4	1,216	389.0
2022	1,104	220.8	1,159	231.9	1,217	243.3	1,276	255.3
Depreciation in 2022		1,065.9		1,100.3		1,135.7		1,172.1
Capex in 2022		1,104.1		1,159.3		1,216.7		1,276.3
Difference		38.2		59.0		81.0		104.2
Capex as % of Depreciation		103.6%		105.4%		107.1%		108.9%

EFFECT OF 15-YEAR DEPRECIATION

- With a 15-year depreciable life, capex is always materially greater than depreciation

Excess of Capital Expenditures over Depreciation, Assuming 15-year Life with No Residual Value



A SUMMARY TABLE

- The table below summarizes the relationships between capex and depreciation for different lives, growth rates, and depreciation methods (zero residual value)

Excess of Capital Expenditures Over Depreciation					
	Depreciation Method	Growth rate:			
		2%	3%	4%	5%
5 year life	Straight line	5.03%	7.56%	10.11%	12.67%
	Double declining	3.58%	5.36%	7.13%	8.89%
	Sum of the digits	3.66%	5.49%	7.31%	9.12%
10 year life	Straight line	10.22%	15.50%	20.87%	26.35%
	Double declining	7.73%	11.62%	15.52%	19.43%
	Sum of the digits	7.05%	10.60%	14.17%	17.76%
15 year life	Straight line	15.58%	23.79%	32.27%	40.99%
	Double declining	11.95%	18.03%	24.16%	30.34%
	Sum of the digits	10.48%	15.83%	21.24%	26.69%

Courts Generally Have Accepted Capex $\geq$ Depreciation

- Unfortunately, some federal and Delaware court decisions have accepted DCF valuations in which depreciation equaled capital expenditures
- Other federal and Delaware court decisions have accepted DCF valuations in which depreciation exceeded capital expenditures
  - Two Delaware decisions have accepted DCF valuations where capital expenditures were ***less than half*** of depreciation!

The Appropriate Treatment of Amortization in DCF Valuations

AMORTIZATION

- Amortization and depreciation are both *non-cash* charges that reduce reported income

- Tax-deductible amortization is similar to depreciation in that it reduces both reported net income and taxes
  - Non-tax-deductible amortization reduces only net income
  - Most amortizable intangible assets are created through either acquisitions or creation of intellectual property

Amortization Has a Limited Life

- An important difference between amortization and depreciation must be recognized by valuators when calculating terminal value: **amortization has a limited life**

- A common error is to project growth in amortization in perpetuity

  - Amortizable intangible assets such as goodwill are not systematically replaced in the ordinary course of business

  - Since amortization, unlike depreciation, does not grow in perpetuity, it must be separately valued in terminal value calculations

Amortization Must Be Separated from Depreciation in D&A

- Companies customarily report depreciation and amortization ("D&A") as a single line item in their income and cash flow statements
- Because of the substantive differences between amortization and depreciation, it is important that valuators determine how much of the projected D&A is amortization

The Value of Amortization Is the Present Value of Future Tax Benefits

- Even though amortization should be excluded from the computation of terminal value, any tax benefit it generates has value and should be included in enterprise value

- An appropriate manner to value amortization subsequent to the projection period is to determine the risk-adjusted present value of the future tax benefits of the remaining amortization

Other Normalizing Adjustments

- FCF must be also be normalized to exclude any other items that are not growing over time or which have a finite term, such as tax-loss carryforwards, limited-life royalties, and non-compete agreements
- The present value of future positive or negative cash flows from limited-life items after the projection period should be included in terminal value
  - The value of tax-loss carryforwards is the risk-adjusted present value of future tax benefits
  - The value of future limited-life income streams is the present value of the income net of taxes
  - The value of future limited-life obligations is the negative present value of the expense net of taxes

A Simple Formula

- These adjustments are achieved by adding the present value of these net cash flows after the terminal year to enterprise value, as shown in the following equation:

$$EV = PV_F + PV_T + PV_A$$

EV = enterprise value at the valuation date;

PV_F = present value of free cash flows from the valuation date through the terminal year of the projection;

PV_T = present value of terminal value based on normalized FCF

PV_A = present value of net benefits (costs) of amortization, tax-loss carryforwards, and limited-life income and expense items after the terminal year of the projection

Erroneous Treatment of Amortization by Experts in Court

- An example of the erroneous treatment of amortization in a DCF analysis is a 2007 Delaware decision in which annual tax-deductible amortization of $5.4 million was included as a non-cash charge in the Court's valuation model
- Since amortization was part of the projected free cash flow that the testifying experts used in their growth models, they effectively assumed that the amortization was perpetual, leading to an overstated valuation by the Court

SUMMARY – D&A, CAPEX & TERMINAL VALUE

- As a general rule, capital expenditures should be greater than depreciation in a terminal value calculation
- The relationship is a function of depreciation rates, company growth rates and technological innovation
- Amortization of intangible assets, loss carryforwards, and other limited-life assets (and liabilities) should be excluded from normalized FCF in terminal value and should be separately valued
- Since data supplied by management often lumps depreciation and amortization together, the valuator must obtain the granular information necessary for an appropriate analysis

I would like to thank

Prof. Mauro Bini

for the opportunity

to share my ideas with you

at this 5th Annual International

Conference of the OIV

Your questions and comments are welcome

Sample Calculations of Relationship between Capital Expenditures and Depreciation

3% Growth – 15 Year Straight Line Depreciation								
Year	Capital Expenditures	2027	2028	2029	2030	2031	2032	2033
2017	1,000.0	50.0						
2018	1,030.0	103.0	51.5					
2019	1,060.9	106.1	106.1	53.0				
2020	1,092.7	109.3	109.3	109.3	54.6			
2021	1,125.5	112.6	112.6	112.6	112.6	56.3		
2022	1,159.3	115.9	115.9	115.9	115.9	115.9	58.0	
2023	1,194.1	119.4	119.4	119.4	119.4	119.4	119.4	59.7
2024	1,229.9	123.0	123.0	123.0	123.0	123.0	123.0	123.0
2025	1,266.8	126.7	126.7	126.7	126.7	126.7	126.7	126.7
2026	1,304.8	130.5	130.5	130.5	130.5	130.5	130.5	130.5
2027	1,343.9	67.2	134.4	134.4	134.4	134.4	134.4	134.4
2028	1,384.2		69.2	138.4	138.4	138.4	138.4	138.4
2029	1,425.8			71.3	142.6	142.6	142.6	142.6
2030	1,468.5				73.4	146.9	146.9	146.9
2031	1,512.6					75.6	151.3	151.3
2032	1,558.0						77.9	155.8
2033	1,604.7							80.2
Annual Depreciation		**1,163.6**	**1,198.5**	**1,234.4**	**1,271.5**	**1,309.6**	**1,348.9**	**1,389.4**
Capital Expenditures		**1,343.9**	**1,384.2**	**1,425.8**	**1,468.5**	**1,512.6**	**1,558.0**	**1,604.7**
Capital Expenditures in Excess of Depreciation		**180.3**	**185.7**	**191.3**	**197.1**	**203.0**	**209.1**	**215.3**
Difference in %		**15.50%**	**15.50%**	**15.50%**	**15.50%**	**15.50%**	**15.50%**	**15.50%**

3% Growth – 15 Year Double Declining Depreciation								
Year	Capital Expenditures	2027	2028	2029	2030	2031	2032	2033
2017	1,000.0	32.8						
2018	1,030.0	67.5	33.8					
2019	1,060.9	69.5	69.5	34.8				
2020	1,092.7	71.6	71.6	71.6	35.8			
2021	1,125.5	73.8	73.8	73.8	73.8	36.9		
2022	1,159.3	85.5	76.0	76.0	76.0	76.0	38.0	
2023	1,194.1	110.0	88.0	78.3	78.3	78.3	78.3	39.1
2024	1,229.9	141.7	113.3	90.7	80.6	80.6	80.6	80.6
2025	1,266.8	182.4	145.9	116.7	93.4	83.0	83.0	83.0
2026	1,304.8	234.9	187.9	150.3	120.2	96.2	85.5	85.5
2027	1,343.9	134.4	241.9	193.5	154.8	123.9	99.1	88.1
2028	1,384.2		138.4	249.2	199.3	159.5	127.6	102.1
2029	1,425.8			142.6	256.6	205.3	164.2	131.4
2030	1,468.5				146.9	264.3	211.5	169.2
2031	1,512.6					151.3	272.3	217.8
2032	1,558.0						155.8	280.4
2033	1,604.7							160.5
Annual Depreciation		**1,204.10**	**1,240.10**	**1,277.50**	**1,315.70**	**1,355.30**	**1,395.90**	**1,437.70**
Capital Expenditures		**1,343.90**	**1,384.20**	**1,425.80**	**1,468.50**	**1,512.60**	**1,558.00**	**1,604.70**
Capital Expenditures in Excess of Depreciation		**139.80**	**144.10**	**148.30**	**152.80**	**157.30**	**162.10**	**167.00**
Difference in %		**11.62%**	**11.62%**	**11.62%**	**11.62%**	**11.62%**	**11.62%**	**11.62%**

3% Growth – 15 Year Sum-of-the-Digits Depreciation								
Year	Capital Expenditures	2027	2028	2029	2030	2031	2032	2033
2017	1,000.0	9.1						
2018	1,030.0	28.1	9.4					
2019	1,060.9	48.2	28.9	9.6				
2020	1,092.7	69.5	49.7	29.8	9.9			
2021	1,125.5	92.1	71.6	51.2	30.7	10.2		
2022	1,159.3	115.9	94.8	73.8	52.7	31.6	10.5	
2023	1,194.1	141.1	119.4	97.7	76.0	54.3	32.6	10.9
2024	1,229.9	167.7	145.3	123.0	100.6	78.3	55.9	33.5
2025	1,266.8	195.8	172.7	149.7	126.7	103.6	80.6	57.6
2026	1,304.8	225.4	201.6	177.9	154.2	130.5	106.8	83.0
2027	1,343.9	122.2	232.1	207.7	183.3	158.8	134.4	110.0
2028	1,384.2		125.8	239.1	213.9	188.8	163.6	138.4
2029	1,425.8			129.6	246.3	220.3	194.4	168.5
2030	1,468.5				133.5	253.7	227.0	200.3
2031	1,512.6					137.5	261.3	233.8
2032	1,558.0						141.6	269.1
2033	1,604.7							145.9
Annual Depreciation		**1,215.1**	**1,251.6**	**1,289.1**	**1,327.8**	**1,367.6**	**1,408.6**	**1,450.9**
Capital Expenditures		**1,343.9**	**1,384.2**	**1,425.8**	**1,468.5**	**1,512.6**	**1,558.0**	**1,604.7**
Capital Expenditures in Excess of Depreciation		**128.8**	**132.7**	**136.7**	**140.8**	**145.0**	**149.3**	**153.8**
Difference in %		**10.60%**	**10.60%**	**10.60%**	**10.60%**	**10.60%**	**10.60%**	**10.60%**

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